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Do Maxicharged Particles Exist?

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The critical charge Z_c is estimated for elementary particles using a Newton–Wigner position operator-inspired model. Particles with $Z \sim Z_c$ (maxicharged particles), if they exist at all, can have unusual properties which make them illusive objects, that are not easy to detect. Dirac's magnetic poles have a (magnetic) charge $g \gg Z_c$. This gives one more argument that it is unexpected for pointlike monopoles to be found in our world, where $\alpha^{-1} = 137$.

The aim of this note is to raise a question, rather than to give answer it. Why do all observed elementary (not composite) particles have small electric charge $|Z| \leq 1$? May elementary particles with $|Z| > 1$ exist?

This question can be considered as an aspect of the charge quantization mystery. Although this quantization can be understood in the framework of grand unification theories(1) or even in the standard model,(2) the most elegant explanation dates back to Dirac's seminal paper(3),3 on magnetic monopoles. None of these approaches actually exclude the existence of multicharged particles.

As small electric charges can more easily escape detection than big charges, theorists are more willing to introduce the former in their theories. So in the literature such exotic creatures can be found as millicharged(5) or minicharged(6) particles. They have been sought experimentally,(7) but not yet found. As for multicharged particles, only a few (to our knowledge) examples have been suggested. A doubly charged Higgs boson was introduced in refs. 8 and a doubly charged (but composite) lepton in refs. 9 Neither of them has been found as yet.(10)

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[3] For the inclusion of fractionally charged particles see, for example, ref. 4.

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At least one reason can be imagined which makes big charges uncomfortable. It is well known[11,12] that when the charge on a nucleus increases, the ground-state electron energy level in its Coulomb field lowered and for some critical value of the charge, $Z_c \simeq 170$, plunges into the Dirac sea of negative energy levels. After this the vacuum becomes unstable. So Z_c determines an "electrodynamic upper frontier" for the periodic system of chemical elements.

But a finite size of the nucleus, which removes the Coulomb field singularity at the origin, plays an important role in reaching such a conclusion and in the calculation of Z_c: The Dirac equation with bare Coulomb potential becomes ill-defined for $Z > 137$. And fundamental elementary particles (quarks, leptons, . . .) are believed to be pointlike. So at first sight the above-described notion of critical charge does not make sense for them.

However, an arbitrarily precise localization is impossible for a relativistic particle, as was realized a long time ago.[13] This means that in relativistic theory an elementary particle no longer can be considered as a pointlike source for the Coulomb field.

The meaning of localization for relativistic particles has been carefully investigated.[14,15] In particular, the most localized wavepacket for a spin-zero particle with mass m, which does not contain any admixture of negative frequencies, is given by the Newton–Wigner wave function[14]

$$\psi(r) \sim \left(\frac{m}{r}\right)^{5/4} K_{5/4}(mr) \tag{1}$$

where $K_\nu(r)$ is a modified Bessel function.

Unfortunately, $\psi(r)$ in (1), belonging to the continuous spectrum, is not normalizable and diverges at the origin as $r^{-5/2}$. But it cannot be expected that the one-particle picture which is assumed in (1) remains valid for distances $r \ll m^{-1}$. Therefore, we may consider the following simple model for a pointlike elementary particle with electric charge Ze:

$$\tilde{\rho}(2) = (Ze)^{-1}\,\rho(r) = \begin{cases} 0 & \text{if} \quad r \leq r_0 \\ Cr^{-5/2}K^2_{5/4}(mr) & \text{if} \quad r > r_0 \end{cases} \tag{2}$$

Here $\rho(r)$ stands for the charge density at a point $\mathbf{r}$, and the constant C is determined from the normalization condition

$$4\pi \int_0^\infty \rho(r) r^2\, dr = Ze \tag{3}$$

The cutoff parameter r_0 must obey $r_0 \ll m^{-1}$. We have somewhat arbitrarily take $r_0 = 0.01m^{-1}$. The prescription $\rho = 0$ when $r \leq r_0$ is a reflection of our desire to have (2) resemble the topological soliton model for the electron.[(16),4] Instead we may take $\rho(r) = \text{const} \equiv \rho(r_0)$ for $r \leq r_0$: the results do not change significantly for massive enough particles, and for the lightest particle, still in the realm of our interest, the difference does not exceed 15%. Having in mind the qualitative nature of our argumentation, such subtleties will be set aside. Note that in ref. 16 r_0 coincides with the classical electron radius $(137m)^{-1}$, giving some justification for our choice. If a charge e_1 probes the spherically symmetric charge distribution (2), the potential energy of their interaction is

$$V = -4\pi\alpha \left[\frac{1}{r}\int_0^r x^2\,\tilde{\rho}(x)\,dx + \int_r^\infty x\tilde{\rho}(x)\,dx\right] \tag{4}$$

where $\alpha = Z|ee_1|/4\pi$, and charges of opposite sign were assumed.

Now we consider Dirac's equation, with the potential defined from (2)–(4), for the group-state energy level in the situation when this level just enters the negative energy sea, that is, $E = -1$, in units for which the probe particle mass $m_1 = 1$. For $m \gg m_1$, this equation for the radial function G is[(11)]

$$\ddot{G} - \frac{\dot{V}}{V}\dot{G} + \left[V(V+2) + \frac{1}{r}\frac{\dot{V}}{V}\right]G = 0 \tag{5}$$

where dots designate derivatives, for example, $\dot{G} = dG/dr$.

By the substitution $G(r) = \sqrt{V(r)}\psi(r)$, this equation takes a form which is more convenient for numerical calculations:

$$\ddot{\psi} + \left[V(V+2) + \frac{1}{r}\frac{\dot{V}}{V} + \frac{\ddot{V}}{2V} - \frac{3}{4}\left(\frac{\dot{V}}{V}\right)^2\right]\psi = 0 \tag{6}$$

For large distances $r \gg m^{-1}$, $K^2_{5/4}(mr)$ in (2) falls as e^{-2mr}. Therefore the second term in (4) can be dropped for such distances and the first term, because of the normalization condition (3), gives just the Coulomb potential $V(r) = -\alpha/r$, for which equation (5) is exactly solvable in terms of the modified Bessel function of complex index[(11)]

$$G(r) \sim K_{i\nu}\,(\sqrt{8\alpha r}), \qquad \nu = 2\sqrt{\alpha^2 - 1} \tag{7}$$

Let us take some $R \gg (2m)^{-1}$. Equation (5) [in fact (6)] can be numerically solved in the region $0 \leq r \leq R$ subject to the boundary conditions $G(0) = 0$, $\dot{G}(0) \neq 0$. Then the smoothness of the logarithmic derivative at $r = R$ gives an equation which determines the critical coupling α_c:

[4] For another approach see ref. 17.

$$\left.\frac{z\, dK_{i\nu}(z)/dz}{K_{i\nu}(z)}\right|_{z=\sqrt{8\alpha r}} = \left.\frac{2R\dot{G}(r)}{G(r)}\right|_{r=R} \tag{8}$$

The critical coupling so evaluated shows a weak dependence on the mass m and changes from $\alpha_c \simeq 1.03$ for $m = 10^4$ to $\alpha_c \simeq 1.1$ for $m = 20$. These numbers correspond to the choice $R = 10m^{-1}$. If we take $R = 5m^{-1}$ instead, the modifications do not exceed a few percent. Roughly modeling the particle–antiparticle situation by setting $m = 2$, we find $\alpha_c \approx 2.5$.

We infer the following main conclusion from the above considerations: every pointlike electric charge Ze such that $Z^2e^2/4\pi \approx Z^2/137 > 2$–$3$ destabilizes the vacuum.

The actual value of α_c can be even smaller if we remember that field-theoretic effects decrease Z_c in the case of the nucleus[18] and some investigations show that a chiral phase transition is expected in strongly coupled QED for $\alpha_c \approx \pi/3$.[19],5

In any case in the following we will treat $\alpha_c \approx 2$–3 as a fair estimate. So $Z_c \approx 15$–20 can be considered as an "electrodynamic upper frontier" for pointlike elementary particles.

But there is quite a lot space from 1 to Z_c. Where are the particles inhabiting this interval?

Particles with $\alpha \approx Z^2/137 > 1$ (we will call them maxicharged particles) are of particular interest because their interactions are essentially nonperturbative. For example, an "onium" from such a particle and antiparticle will decay more readily into $(n + 1)$ photons than into n photons because now $Ze > 1$. This means that in fact it decays into an infinite number of soft photons, that is, into a classical field.

Another remarkable property of the maxicharged particles is that their classical radius $r_0 = \alpha/m$ ($\alpha \approx Z^2/137$) is bigger than their quantum size (Compton wavelength) $\lambda = 1/m$. Because of this property it is not very easy to produce them in, for example, electron–positron collisions. If $\tau \sim 1/m$ is the production time of a maxicharged particle–antiparticle pair and τ_0 their annihilation time, then[21]

$$\frac{\tau}{\tau_0} \sim \alpha\left(\frac{\lambda}{r_0}\right)^3 = \alpha^{-2} < 1$$

So the pair is annihilated before they are created[21]! This suggests that maxicharged particles can be rather illusive objects, irrespective of their masses.

In fact, the notion of maxicharged particles was introduced by Schwinger.[22] Below we repeat his arguments from which a more clearly defined notion of maxicharged particles can be deduced.

[5] For a recent discussion see, for example, ref. 20.

Electrodynamics with electric charges e, and magnetic charges g reveals a duality symmetry which can be viewed as a rotation in the (e, g) space. However, this symmetry should be spontaneously violated,[23] that is, we should have a definite direction for the electric axis in the (e, g) space. In fact this direction can be guessed from the fact that only small charges surround us in our world.[22] First of all, let us introduce an invariant definition of small charges[22]: we will say that a particle with electric charge e_a and magnetic charge g_a belongs to the category of small charges if

$$\frac{e_a^2 + g_a^2}{4\pi} < 1 \tag{9}$$

Correspondingly big charges (maxicharged particles in our terminology) are defined through

$$\frac{e_a^2 + g_a^2}{4\pi} \geq 1 \tag{10}$$

If a and b are an arbitrary pair of small charges, then

$$\left(\frac{e_a g_b - e_b g_a}{4\pi}\right)^2 \leq \frac{e_a^2 + g_a^2}{4\pi} \frac{e_b^2 + g_b^2}{4\pi} < 1 \tag{11}$$

On the other hand, Schwinger's symmetrical quantization condition reads:

$$\frac{e_a g_b - e_b g_a}{4\pi} = n \tag{12}$$

where n is an integer.

Now (11) and (12) are compatible only if $n = 0$! Therefore, for any pair of small charges we have[22]

$$\frac{g_a}{e_a} = \frac{g_b}{e_b}$$

This means that small charges occupy a single line in the (e, g) space, and it seems from our everyday experience that just this line is chosen as representing the electric charge axis after spontaneous breakdown of the duality symmetry. In other words, none of the small charges possess any amount of magnetic charge. Dyons can live only in the wonderland of maxicharged particles!

Now we turn to a more speculative line of reasoning. The most natural symmetrical solution of Dirac's (nonsymmetrical) quantization condition

$$\frac{eg}{4\pi} = \frac{n}{2}, \qquad n \text{ an integer}$$

would be $e = g$. So in such a hypothetical world singly charged particles will have $\alpha = e^2/4\pi = 0.5$, and doubly charged particles would have $\alpha = 2$. Clearly triply charged particles lie beyond the vacuum stability border if we adopt the above-cited value for the critical coupling $\alpha_c \simeq 2$–3. In fact even doubly charged particles look suspicious enough. So maybe the absence of multicharged particles is mere reminiscent of the epoch when there was a full harmony between electrical and magnetic forces?

Note that for the above picture to have any chance to be valid, something must happen to the *scale* in the duality space, not only to the orientation of the electric axis, because we know quite well that $\alpha \simeq (137)^{-1}$ and not 0.5! Can we hope that the present value of the fine structure constant is associated with the symmetry breaking between electric and magnetic forces and so can be understood from purely symmetry considerations? Here we have a tempting association that just from conformal (or scale) symmetry considerations Wyler obtained his marvelous formula[24]

$$\alpha = \frac{9}{16\pi^3}\left(\frac{\pi}{5!}\right)^{1/4} \simeq \frac{1}{137.03608}$$

(For discussions of this formula, see refs. 25. Some different "derivations" of this or similar formulas can be found in refs. 26, and for other attempts to calculate the fine structure constant, see refs. 27.)

Maybe this "number in search of a theory"[28] at last finds it in electromagnetic duality and its breaking?

Russian folklore says that "one simpleton can ask so much questions that hundred sages fail to answer." We hope that the questions raised in this note do not fall into such a category.

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